Math 1060

Lecture 1
Introduction & Exponential Functions

Outline

Introduction to Math 1060

Math 1060 is the first course in the three-course calculus sequence at Clemson.

Math 1060: Limits, derivatives, and basic integrals
Math 1080: More advanced integrals
Math 2060: Calculus in several variables; vector calculus

Instructor information & web site

Dr. Charles (Chris) Johnson, ccjohns@clemson.edu
Martin O-06

Office hours are 1:30 - 2:10pm on Monday, Wednesday, and Friday; and 2:00 - 3:15pm on Thursdays.

Other times by appointment only.

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Course web sites: http://ccjohnson.org/math1060
http://math.clemson.edu
http://bb.clemson.edu
http://webassign.net

Coordinated courses

Math 1060 at Clemson is a *coordinated course*, meaning all sections take the same test on the same day at the same time. All exams are graded the same way, and all final grades are assigned the same way.

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50 minute lectures	20 minute lectures

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50 minute lectures	20 minute lectures
Homeworks & quizzes	Daily in-class learning activities

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50 minute lectures	20 minute lectures
Homeworks & quizzes	Daily in-class learning activities

This section will use the traditional model.

Exams

Each of the three regular semester exams counts for 15% of your final grade, and will be at 6:45 - 8:15pm.

Wednesday September 24
Wednesday October 22
Wednesday November 19.

The final exam will count 25% of your final grade and will be Monday, December 8 at 11:30am - 2:00pm. The final exam replaces your lowest exam grade, if it helps your grade.

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Calculators are not allowed on the exams! No make-up exams!

Accommodations for students with disabilities

If you have a letter from Student Disability Services which gives you accommodations during an exam, you must turn the letter into the instructor at least one week before the first exam!

Homeworks

Two types of homework:

1. Online homework completed with WebAssign (10% of final grade).

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Late assignments will not be accepted!

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There will not be any make-up quizzes!

ALEKS

You will be required to complete a pre-calculus module in the online ALEKS system. If you do not receive a mastery level of 85% in ALEKS by the first exam, your final grade will be lowered by one full letter grade.

Grading summary

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- Long quizzes announced a few days in advance, and short pop quizzes randomly. Quiz average is 10% of your grade.
- Final letter grade assigned on traditional 10-point scale.
- Failure to reach 85% in ALEKS lowers your final grade by one letter.

How to get a good grade

Everyone is capable of getting an A in this course, but most people won't work hard enough for an A. Here are some tips to help you get the best grade you can.

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4. Don't get cocky and think you understand more than you do.
5. Don't complain that you don't have time: make time.

Natural numbers and integers

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Notice every natural number is an integer, but not every integer is a natural number.

Some notation

We use the symbol $\in$ to mean something is an element is in a set, and the symbol $\notin$ to mean something is not an element of a set.

For example,

$$-7 \in \mathbb{Z},$$

but

$$-7 \notin \mathbb{N}.$$

Rational numbers

The collection of all *rational numbers*, that is numbers which may be written as a fraction

$$\frac{p}{q}$$

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Notice that every integer (and hence every natural number) is also a rational number: just take the denominator to be 1,

$$3 = \frac{3}{1}$$

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Every rational number (and so every integer and every natural number) is real, but there are real numbers which are not rational.

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$$\sqrt{2} = 1.41417...$$

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Numbers which are real, but not rational, are called *irrational numbers*.

Exponentials

An *exponential function* is a function of the form $f(x) = a^x$ where $a > 0$ is some real number called the *base*. If our input to the function is a natural number, we know what this should be:

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$$

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ times}}$$

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How should we define a^x for a real number x?
We'll work our way up, defining zero and negative exponents, then rational exponents.

Zero exponents

Notice that if $m, n \in \mathbb{N}$, then $a^m \cdot a^n = a^{m+n}$:

$$a^m \cdot a^n = \underbrace{a \cdots a}_{m \text{ times}} \cdot \underbrace{a \cdots a}_{n \text{ times}} = \underbrace{a \cdots a}_{m+n \text{ times}} = a^{m+n}.$$

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Thus $a^{-n} = \frac{1}{a^n}$.

Powers of exponents

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Fractional powers

Now we can define exponents of fractional powers of exponents:

$$a = a^1 = a^{n/n} = a^{n \cdot 1/n} = \left(a^{1/n}\right)^n.$$

As raising $a^{1/n}$ to the n-th power equals a, we must have that $a^{1/n}$ is the n-th root of a: $a^{1/n} = \sqrt[n]{a}$.

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Note this can cause some trouble if we try to take an even root of a negative number: $(-1)^{1/2} = \sqrt{-1}$ is not a real number! In this class we will only use real numbers: if you get a complex number as an answer (i.e., something that involves $\sqrt{-1}$), then you've made a mistake.

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$$0.125^{4/3} = \left(0.125^{1/3}\right)^{4} = 0.5^{4} = 0.0625$$

Irrational powers

We now know how to define exponents for real numbers which are natural numbers, integers, and rational numbers, but we don't yet know how to define irrational powers.

To actually answer this question "properly" requires calculus, but the basic idea is very easy.

We will explain the idea by way of an example: calculating 2^π.

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1. Consider a *sequence* of rational numbers which approaches π. For example,

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   Each one of these is just a rational power, so something we know how to to calculate. The decimal values are

$$8, 8.5741877\ldots, 8.8152409\ldots, 8.82135330\ldots, 8.8244110\ldots,$$

Irrational powers

3. Take the *limit* of this sequence (this is where the calculus comes in). Intuitively, the values

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This idea works for any irrational power x in calculating a^x.

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For this reason it is best to *avoid decimal approximations whenever possible*! You should leave answers in terms of π and $\sqrt{17}$ and $42\sqrt[3]{7/5}$ instead of trying to approximate them with decimals.

Properties of exponents

The following theorem summarizes what we've discussed above:

Theorem

For all positive real numbers $a > 0$ *and* $b > 0$, *and for every pair of real numbers* x *and* y, *the following five properties hold:*

(i) $a^0 = 1$

(ii) $a^x \cdot a^y = a^{x+y}$

(iii) $\frac{a^x}{a^y} = a^{x-y}$

(iv) $(a^x)^y = a^{xy}$

(v) $(ab)^x = a^x \cdot b^x$

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(iii) $\frac{a^x}{a^y} = a^{x-y}$

(iv) $(a^x)^y = a^{xy}$

(v) $(ab)^x = a^x \cdot b^x$

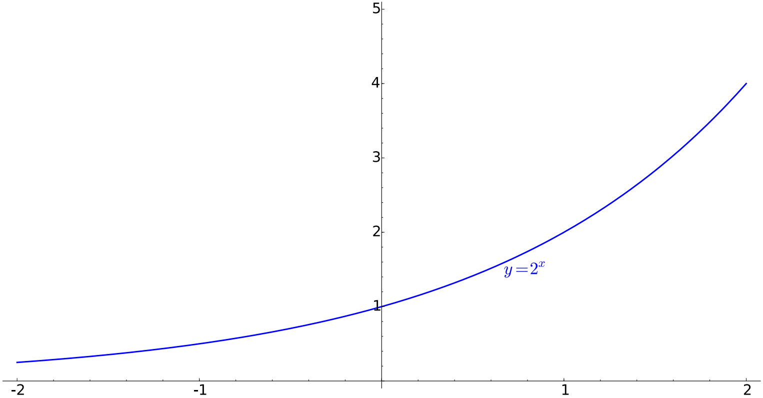
We described properties (*i*), (*ii*), and (*iii*) above. Properties (*ii*) and (*iv*) follow by similar reasoning. You should spend a few minutes outside of class thinking about the logic behind (*ii*) and (*iv*).

Graphs of exponentials

Suppose for the moment $a > 1$. Then the function $f(x) = a^x$ is *increasing*: this means that if $x_2 > x_1$, then $a^{x_2} > a^{x_1}$. There are lots of increasing functions, but exponentials are special because they grow *very* quickly, as indicated by the graph $y = a^x$.

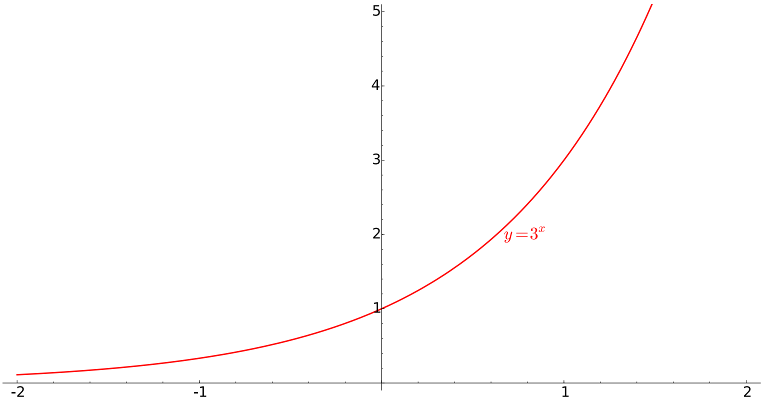
Let's look at a few graphs of these exponentials and see if we notice any common features.

Graphs of exponentials



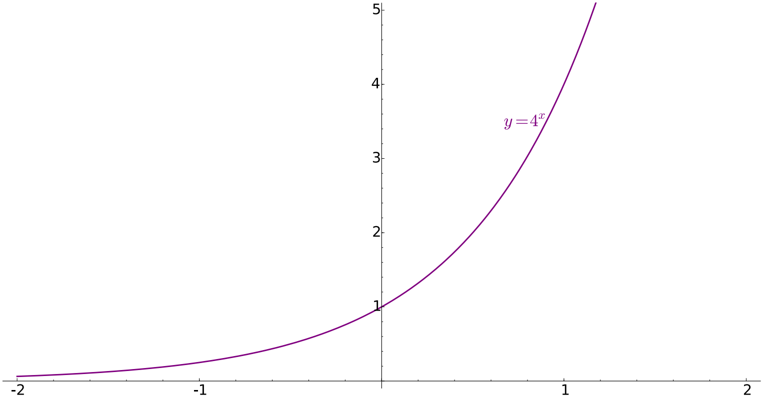
The graph of the exponential function 2^x.

Graphs of exponentials



The graph of the exponential function 3^x.

Graphs of exponentials



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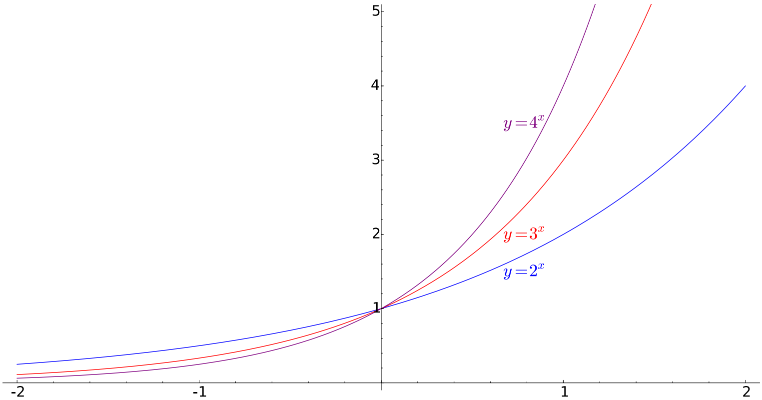


The three previous exponential functions plotted together.

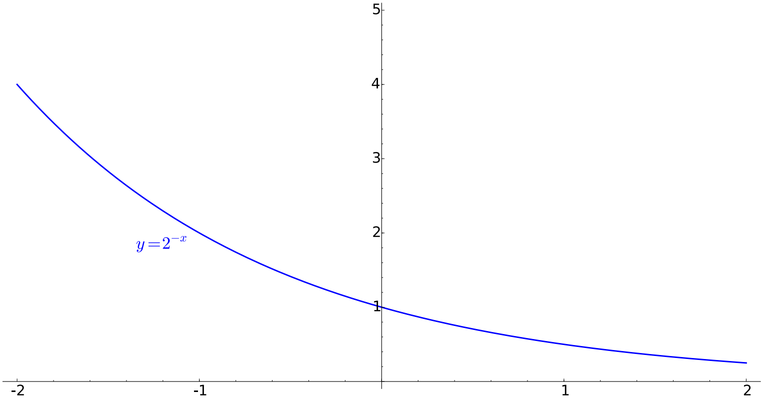
Common features

Let's notice some common features of these three graphs:

1. Each graph passes through the point $(0, 1)$.
2. As x gets big, a^x gets very big very quickly.
3. The bigger a is, the bigger a^x is if $x > 0$.
4. The bigger a is, the smaller a^x is if $x < 0$.
5. $a^x > 0$ for all x

Graphs of negative exponentials

Let's repeat the same procedure graphing $y = a^{-x}$.



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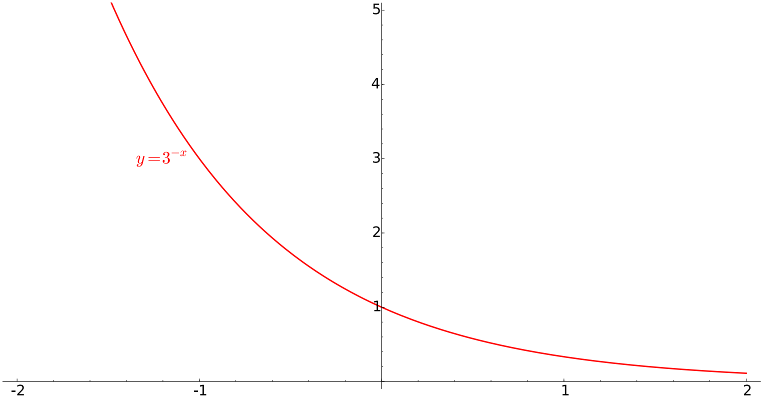

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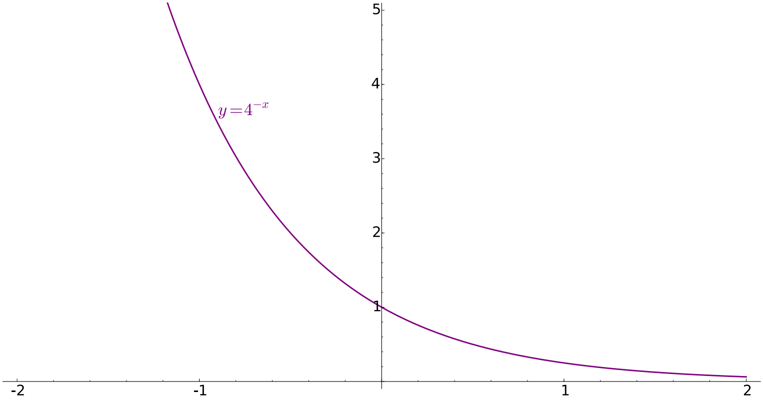

The graph of the exponential function 4^{-x}.

Graphs of negative exponentials

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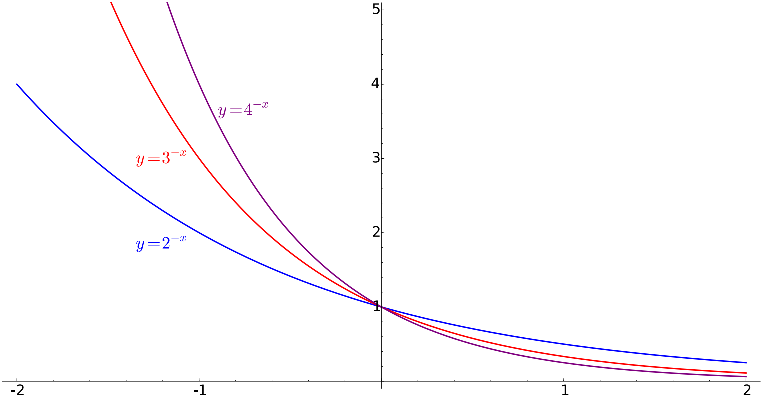

The three previous exponential functions plotted together.

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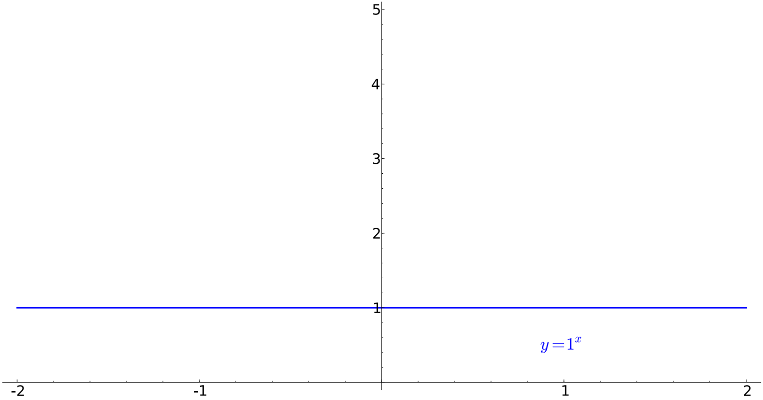
1^x

Note 1^x is simply a horizontal line.



$0 < a < 1$

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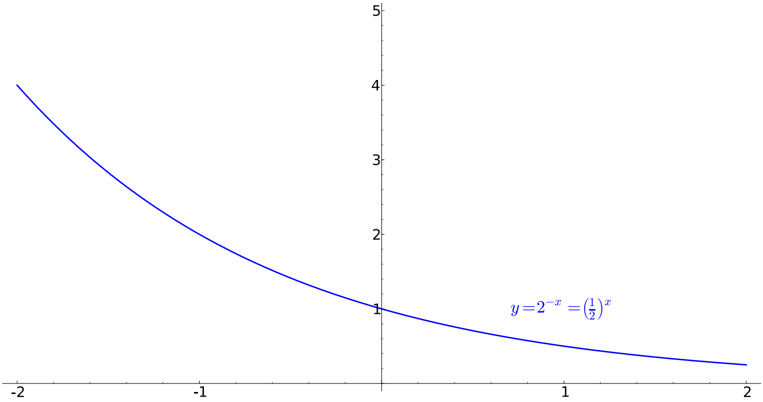
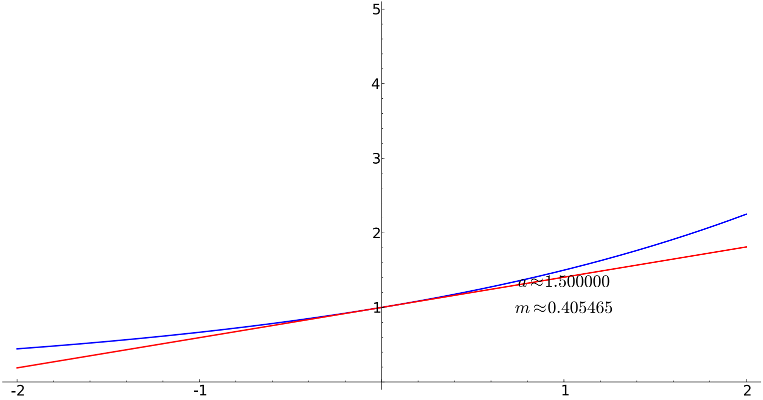
Note that if $0 < a < 1$, then $a^{-1} = {}^1\!/_a > 1$ and $a^x = \left(a^{-1}\right)^{-x} = ({}^1\!/_a)^{-x}$, and this is a case we already understand.



Tangent lines

For each of the graphs of $y = a^x$, let's consider the line that passes through the point $(0, 1)$ and is tangent to the graph.



The number e

By modifying the value of a, we can make the slope of this line any positive value we want.

The value of a which makes the slope equal to 1 is called *Euler's constant* and is denoted by the letter e. The first few digits of the decimal expansion of e are $e = 2.71828\ldots$.

Even though this number seems weird and random right now, it actually number comes up *all the time* in calculus. To explain why this number is important and comes up all the time is a little bit of a long story, but you will learn bits of that story throughout your calculus courses.

Homework

1. **Due Monday, 8/25**:
   - Read Ch. 1 of Stewart
   - Stewart §1.5: 2, 4, 7, 15
   - Stewart §1.6: 5 - 8, 29, 30

Summary

- Described coordinated courses, exams, homeworks, quizzes, and grading.
- Defined four common sets of numbers: $\mathbb{N}$ (the natural numbers), $\mathbb{Z}$ (the integers), $\mathbb{Q}$ (the rational numbers), and $\mathbb{R}$ (the real numbers).
- Described exponential functions, working our way up from a^n when $n \in \mathbb{N}$ through a^x when $x \in \mathbb{R}$.
- Described the graphs of the functions a^x and a^{-x}, and noticed some commonalities in these graphs.
- Defined the number e as the unique number so that the tangent line to the graph $y = e^x$ at $(0, 1)$ has slope 1.