MATH 1060

LECTURE 9
THE POWER RULE, POLYNOMIALS, AND EXPONENTIALS

Outline

Summary of last lecture

Derivative laws

- The derivative of a constant function
- The derivative of the identity function
- The power law
- The constant multiple law
- The sum and difference laws
- The general power law
- The derivative of e^x

Homework

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- Proved that a function must be continuous in order to be differentiable.
- Discussed two common obstacles to differentiability: corners and cusps.
- Defined higher-order derivatives.
- Saw several different notations for derivatives.
- Briefly mentioned applications of derivatives within mathematics, physics, computer science, and engineering.

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In particular, by the end of this lecture we will understand how to differentiate polynomials and exponential functions.

We will, however, start off with very simple functions and work our way up to more interesting situations.

The derivative of a constant function

Theorem

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In terms of slopes of tangent lines, all this says is that the tangent lines to graphs of the form $y = c$ always have slope to zero.

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In terms of veocity, if the position of a particle at time t is $f(t) = t$, then its velocity is always $f'(t) = 1$.

The power law

Theorem (The power law)

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Proof.

Let $f(x) = x^n$. We will calculate $f'(a)$ using one of our alternative limit definitions:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

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Now we factor the numerator,

$$x^n - a^n = (x - a) \cdot \left(x^{n-1} + x^{n-2}a + \cdots + xa^{n-2} + a^{n+1}\right).$$

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This can be verified by multiplying out the right-hand side and simplifying.

The power law

Proof (continued).

Now,

$$f'(a) = \lim_{x \to a} \frac{(x-a) \cdot \left(x^{n-1} + x^{n-2}a + \cdots xa^{n-2} + a^{n+1}\right)}{x-a}$$

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$$= \lim_{x \to a} \left(x^{n-1} + x^{n-2}a + \cdots + xa^{n-2} + a^{n-1}\right)$$

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Proof (continued).

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So for every a, $f'(a) = na^{n-1}$ and thus $\frac{d}{dx}x^n = nx^{n-1}$. □

The power law

Examples

- If $f(x) = x^9$, then $f'(x) = 9x^8$.

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- If $f(x) = x^3$, then $\frac{d}{dx} f(x) = 3x^2$.

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- $\frac{d}{dx}x^{13} = 13x^{12}$.

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- If $f(x) = x^9$, then $f'(x) = 9x^8$.
- If $f(x) = x^3$, then $\frac{d}{dx}f(x) = 3x^2$.
- $\frac{d}{dx}x^{13} = 13x^{12}$.
- $\frac{d}{dx}x^4\Big|_{x=2} = 4 \cdot 2^3 = 32$.

The power law

Example: Find the equation of the line tangent to the curve $y = x^3$ at the point $(4, 64)$.

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Solution

We know the line will go through $(x_0, y_0) = (4, 64)$, so if we can find the slope m, the equation of the line will be

$$y - y_0 = m(x - x_0).$$

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To find the slope we differentiate x^3 and then evaluate at $x = 4$. Since $y = x^3$, we have $y' = 3x^2$.

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$$y - 64 = 48(x - 4).$$

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$$y - 64 = 48(x - 4).$$

Or, in slope-intercept form,

$$y = 48x - 128.$$

The constant multiple law

Theorem

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Proof.

$$\begin{aligned}
\frac{d}{dx} cf(x) &= \lim_{h \to 0} \frac{cf(x+h) - cf(x)}{h} \\
&= c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= cf'(x).
\end{aligned}$$

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- $\frac{d}{dx}\pi x^9 = 9\pi x^8$.
- If $y = \frac{-4x^5}{7}$, then $y' = \frac{-20x^4}{7}$.

The sum and difference laws

Theorem

If f and g are differentiable functions, then $f + g$ and $f - g$ are also differentiable with derivatives

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x), \text{ and}$$

$$\frac{d}{dx}(f(x) - g(x)) = f'(x) - g'(x).$$

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Homework. □

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- $\frac{d}{dx}\left(x^2 + 3x\right) = 2x + 3$
- If $y = 15x^4 - 6x^3 + \pi x + \frac{13}{2}$, then $y' = 60x^3 - 18x^2 + \pi$.

The sum and difference laws

The position of a ball thrown straight up from a height of four feet with an initial 30 feet per second is given, in feet above the ground, t seconds after being thrown by

$$r(t) = -16t^2 + 30t + 4.$$

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1. What is the ball's instantaneous velocity at time $t = 3$ seconds?
2. What is the ball's speed at time $t = 3$ seconds?

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We've seen before that instantaneous velocity of an object is the derivative of the object's position. Thus the velocity of the ball at time t is

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$$v(t) = -32t + 30$$

When $t = 3$ seconds, the velocity is $v(3) = -66$ feet per second. Speed is the magnitude (absolute value) of velocity, thus the speed of the ball is 66 feet per second.

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The general power law

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(We will have to wait until we learn about the chain rule next week to prove this, but you can go ahead and use this theorem.)

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- $\frac{d}{dx}x^\pi = \pi x^{\pi - 1}$.

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Examples:

- $\frac{d}{dx}x^\pi = \pi x^{\pi-1}$.
- $\frac{d}{dx}x^e = ex^{e-1}$.

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$$\frac{d}{dx}\sqrt{x}$$

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$$= \frac{1}{2\sqrt{x}}.$$

$$\frac{d}{dx}\sqrt[5]{x} = \frac{d}{dx}x^{1/5}$$

$$= \frac{1}{5}x^{-4/5}$$

$$= \frac{1}{5\sqrt[5]{x^4}}$$

The general power law

We can also differentiate some simple fractions:

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The derivative of e^x

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$$=1.$$

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Proof (continued).

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\end{aligned}$$

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&= e^x.
\end{aligned}$$

□

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- $\frac{d}{dx} e^x \Big|_{x=2} = e^2$

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- If $f(x) = e^x$, then $f'(\ln(3)) = 3$.

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- $\frac{d}{dx} e^x \Big|_{x=2} = e^2$
- If $f(x) = e^x$, then $f'(\ln(3)) = 3$.
- Differentiate the function $-3e^x + 4x^4 - 19\pi x^2 + ex + e$:

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Examples:

- $\frac{d}{dx}e^x\Big|_{x=2} = e^2$
- If $f(x) = e^x$, then $f'(\ln(3)) = 3$.
- Differentiate the function $-3e^x + 4x^4 - 19\pi x^2 + ex + e$:

$$-3e^x + 16x^3 - 38\pi x + e.$$

Homework

Due Thursday, September 18 :

1. Read §3.1 in Stewart.
2. Do the problem set listed online at `http://ccjohnson.org/math1060/homework`
3. Seriously, complete ALEKS if you haven't already.