Some general grading guidelines:

1. Grade the quiz out of 10 points, with each problem being worth one point.
2. If the student makes a minor mistake or two, (e.g., loses a sign or makes an arithmetic error) take off 0.5 points. If they make several small errors (e.g., more than three or four arithmetic mistakes) take off a full point.
3. If a student's handwriting is not legible, or if their work is not coherent, count the entire problem as wrong. Rule of thumb: if you can't easily read what they've written or follow their work, don't give any credit.
4. Take off 0.5 points if the student has "disconnected" statements (e.g., no equals signs).

Comments on specific problems:

Problem 1 : If students do not use the $\implies$ symbol, that is okay. If they use the $\implies$ symbol incorrectly (in place of =, for example), then take off 1/2 point. At the end of their work they should explicitly state $f^{-1}(x) = (x-2)^3 + 1$. If their final answer is $x = (y-2)^3 + 1$, then take off 1/2 a point.

Problem 2 : Take off 1/2 point if the student uses the wrong row of Pascal's triangle. If they magically have the right coefficients but Pascal's triangle appears nowhere on their paper, then give full credit but make a remark on their paper, asking how they know these are the right coefficients.

Problem 5 : If they simply write "32" for their answer, give them full credit, but tell them they should explicitly write out $\lim_{x\to 3}(x^3 + 2x - 1) = 32$.

Problem 8 : If the answer is left as the fraction 27/3, give the student full credit, but make a note on their paper that this equals 9.

Problem 9 : This problem has three parts. If the student misses one part, take off 1/2 point. If they miss two or more parts, take off a full point.

Problem 10 : If the student calculates $\delta = 1/200$ but does not show that $0 < |x-2| < 1/200$ implies $|f(x) - 7| < 1/100$, take off 1/2 point.

Name:
Table:

No electronic devices (phones, calculators, computers, etc.) are allowed during the exam.

Each problem is worth one point.

Problem 1

Determine the inverse of the function $f(x) = \sqrt[3]{x-1} + 2$.

Solution

$$\begin{aligned} & y = \sqrt[3]{x-1} + 2 \\ \implies & y - 2 = \sqrt[3]{x-1} \\ \implies & (y-2)^3 = x - 1 \\ \implies & (y-2)^3 + 1 = x \\ \implies & x = (y-2)^3 + 1 \end{aligned}$$

Thus $f^{-1}(x) = (x-2)^3 + 1$.

Problem 2

Simplify the following expression: $(x+y)^5$.

Solution

Use the binomial theorem, first writing out Pascal's triangle:

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

Now applying the binomial theorem we have

$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5.$$

Problem 3

Express $\log_3(7)$ as a quantity where all logarithms have base 10.

Solution

$$\log_3(7) = \frac{\log_{10}(7)}{\log_{10}(3)}.$$

Problem 4

Simplify the following expression: $\ln\left(e^{3x+1} \cdot (e^x)^2\right) - e^{\ln(4x+2)} + 1$.

Solution

$$\begin{aligned}
&\ln\left(e^{3x+1} \cdot (e^x)^2\right) - e^{\ln(4x+2)} + 1\\
=&\ln\left(e^{3x+1} \cdot e^{2x}\right) - (4x+2) + 1\\
=&\ln\left(e^{3x+1+2x}\right) - 4x - 2 + 1\\
=&\ln\left(e^{5x+1}\right) - 4x - 2 + 1\\
=&5x + 1 - 4x - 1\\
=&x
\end{aligned}$$

Problem 5

Determine the following limit: $\lim\limits_{x\to 3}\left(x^3 + 2x - 1\right)$.

Solution

Since $x^3 + 2x - 1$ is a polynomial, we can calculate the limit by evaluation:

$$\lim_{x\to 3}\left(x^3 + 2x - 1\right) = 32.$$

Problem 6

Determine the following limit: $\lim_{x\to 0} \frac{(7+2x)^2 - 49}{x}$.

Solution

We can not simply plug in $x = 0$ to evaluate this limit because of division by zero. To evaluate this limit we need to first do some algebra:

$$\begin{aligned}
\lim_{x\to 0} \frac{(7+2x)^2 - 49}{x} &= \lim_{x\to 0} \frac{49 + 28x + 4x^2 - 49}{x} \\
&= \lim_{x\to 0} \frac{4x^2 + 28x}{x} \\
&= \lim_{x\to 0} (4x + 28) \\
&= 28.
\end{aligned}$$

Problem 7

Let $f(x)$ be the function described below. What is $\lim_{x\to 1^+} f(x)$?

$$f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ \sqrt{2-x} & \text{if } -1 \le x \le 1 \\ x^3 - 3 & \text{if } 1 < x \end{cases}$$

Solution

Because we are taking a right-hand limit, we are only concerned with x-values which are greater than 1. If $x > 1$, then $f(x) = x^3 - 3$. Since this is a polynomial, we can simply evaluate at $x = 1$ to determine the limit:

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \left(x^3 - 3\right) = -2.$$

Problem 8

Suppose the position of a particle at time t is given by the function

$$f(t) = \frac{t^2 + 2t}{t^2 - 7t + 11}.$$

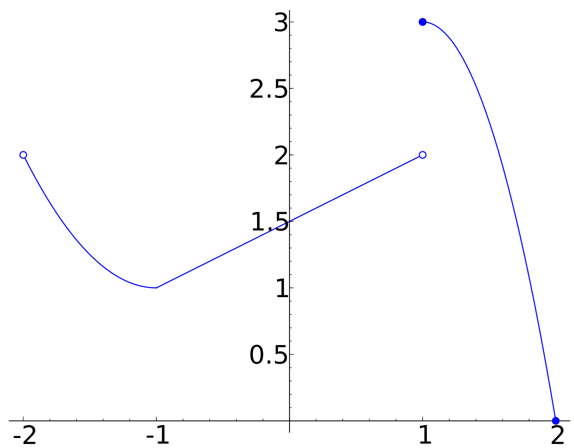
What is the average velocity of the particle over the time interval $[2, 5]$?

Solution

$$\begin{aligned} v_{\text{avg}} =& \frac{f(5) - f(2)}{5 - 2} \\ =& \frac{\left(\frac{5^2+2\cdot5}{5^2-7\cdot5+11}\right) - \left(\frac{2^2+2\cdot2}{2^2-7\cdot2+11}\right)}{5-2} \\ =& \frac{35 - 8}{3} \\ =& \frac{27}{3} \\ =& 9 \end{aligned}$$

Problem 9

Consider the function $f(x)$ whose graph $y = f(x)$ is presented below.



(a) What is the domain of this function?
Solution
$(-2, 2]$

(b) What is the range of this function?
Solution
$[0, 3]$

(c) For what values of a in the domain of the function is the limit $\lim_{x \to a} f(x)$ undefined?
Solution
The limit is defined for every point in the domain of f, except $x = 1$.

Problem 10

Let $f(x) = 2x + 3$. What value of $\delta > 0$ guarantees that $|f(x) - 7| < 1/100$ whenever $0 < |x - 2| < \delta$? For full credit, you must show that your choice of δ does in fact guarantee that $|f(x) - 7| < 1/100$.

Solution

We first work backwards to determine δ:

$$\begin{aligned}
&|f(x) - 7| < 1/100 \\
\implies &|2x + 3 - 7| < 1/100 \\
\implies &|2x - 4| < 1/100 \\
\implies &2|x - 2| < 1/100 \\
\implies &|x - 2| < 1/200.
\end{aligned}$$

It appears that $1/200$ is the correct choice of δ, and now verify this:

$$\begin{aligned}
&0 < |x - 2| < 1/200 \\
\implies &|x - 2| < 1/200 \\
\implies &2|x - 2| < 2 \cdot 1/200 \\
\implies &|2x - 4| < 1/100 \\
\implies &|2x + 3 - 7| < 1/100 \\
\implies &|f(x) - 7| < 1/100.
\end{aligned}$$