Teaching Philosophy

Charles Christopher Johnson

Teaching mathematics effectively is a challenging endeavor that requires engaging students while exposing them to difficult ideas, and also being cognizant of the fact that students from different backgrounds and cultures may have different expectations and desired outcomes than the instructor.

Everyone approaches this issue differently, and my own approach is greatly influenced by my development as a student. I try to emulate the practices of teachers I have admired, while simultaneously motivating students to develop their own understanding of the material discussed in class. To paraphrase Paul Halmos, I want to inspire my students not simply to memorize facts and theorems, but to search for their own questions and answers.

When I taught an introductory number theory course, for example, I had students use Sage to generate lists of Pythagorean triples and come up with their own conjectures before we characterized Pythagorean triples in class. This was an extreme change of pace for most students who had passively attended classes before, but were now being asked to actively develop their own ideas.

Part of my responsibility as a teacher is to engage students and keep them motivated. To maintain students' interest, I like to point out examples showcasing compelling applications of the material discussed in class. When describing normal vectors and tangent planes of surfaces in multivariable calculus, for instance, I show students how these ideas are used in computer graphics to model the way light is reflected off a surface, and this in turn is used to create realistic three-dimensional images on the computer. Even if students are not directly interested in computer graphics, knowing that the topics we discuss have real applications helps them to see the value in those ideas.

While there are several applications that can be discussed in class to keep the material interesting, making class challenging without becoming overwhelming is a more difficult task. This is a balancing act that I am always trying to improve upon, but I do believe it is essential that students are given problems that are not simply *cookie cutter*, but instead require that they exercise their problem-solving skills. To this end I give students regular homework assignments which include some difficult, but not impossible, problems. The intent of these exercises is to have students make connections on their own by combining several concepts which are not explicitly related to one another during lecture.

Though students are sometimes frustrated by difficult problems they can not immediately solve, I make myself available to students to help them, nudge them in the right direction when they get stuck, and tell them that I have faith they can solve the problem if they keep working. I hope that by pushing students to work on harder problems they learn more and are better prepared not only for exams in my class, but are also for more challenging classes they will take later. I remind my students that it is okay to to get stuck on a problem, and it is okay to make mistakes; the important thing is that they persevere and try to learn from the mistakes they do make.

In the future I hope to have the opportunity to work with students on research projects, and by the nature of my research interests I have several ideas for projects that would be appropriate for undergraduates. One possible project would be to consider modifying a *Panov plane*, an object I studied in my thesis, so that it becomes "less symmetric" and asking how much the symmetry can be destroyed while maintaining nice dynamical properties like topological transitivity. This is a question which on the one hand involves a very concrete object that students can study directly without needing an incredible amount of background knowledge, but which also opens the door to a very active and rapidly growing field of research.

In summary, I believe teaching is not simply about relaying facts, but about inspiring and encouraging students to push themselves. This is not always an easy task, but one I pursue by using interesting examples, challenging exercises, and giving regular feedback. While no teacher is perfect, I am continually working towards becoming a better teacher with each passing year.

Addendum: Supporting Evidence

In this addendum I provide some specific examples of how I support students while challenging them with difficult concepts, how I try to address common misconceptions, and how I gradually push students from straight forward exercises to more involved problems.

Supporting Students. The first homework assignment that I assign each semester is to have students locate my office. For each section I teach, I place a sheet of paper on my office door listing each student enrolled in the section as well as a place next to each student's name for their signature, and tell the students their first homework is to come to my office and sign the sheet next to their name. This forces students to physically come to my office at the start of the semester, in the hope that knowing where my office is and coming at least once early on will help students to feel more comfortable coming to my office later in the semester.

This one simple assignment, which the students like because it's an easy homework grade, has noticeably increased the number of students that attend office hours on a regular basis. In the past getting students to come to office hours had been a chore: students would only come in extreme circumstances. Now there is very rarely a day when I do not have students coming to office hours, and they don't come simply for help on assignments. Many students will come after a new theorem or idea has been introduced that they haven't grasped, or come to ask conceptual questions they realized while studying.

Addressing Misconceptions. Many students in calculus struggle with the idea of a limit, and in particular in understanding the difference between taking the limit of a function as x approaches a and evaluating the function at $x = a$. When I anticipate that students will have these types of common misconceptions, I first address them in class through examples, and then give the students homework assignments where they have to face these misconceptions head-on. The following is an example of a homework problem from my calculus class in Fall 2015 at Wake Forest University:

> Construct a pair of functions $f(x)$ and $g(x)$ such that $f(x)$ and $g(x)$ are defined everywhere, and the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist for all a, but
> $$\lim_{x\to 3} f(x)g(x) \neq f(3)g(3).$$

This problem essentially asks students to come up with a function which is discontinuous at $x = 3$, but many students come to office hours initially believing it is impossible to construct a pair of functions as described in the problem. After talking students through the problem, they eventually have an "Ah-ha!" moment when the relationship between a limit and evaluating a function at a point starts to make more sense, and they appreciate why continuous functions are desirable.

Gradual Challenge. There are a variety of reasons why a given student may be averse to challenging themselves, but learning any new or difficult concept inherently requires challenge. In order to help my students build confidence and be more open to challenge, I assign regular exercises that gradually push students a little bit more with each assignment.

After each lecture I assign a fairly brief, straight-forward series of problems that are very comparable to examples in class. I usually try to make these assignments at a level where a good student that has been following along in class and understood the material can finish the assignment within twenty minutes. An example of a problem that was on one of these assignments after covering implicit differentiation is the following:

> Suppose x and y are related by the equation $5x^2 - y^2 = 7$. Use implicit differentiation to find $\frac{dy}{dx}$.

Questions like this are meant to give the students practice with some simple problems, build confidence, and force the students to stay up-to-date with the material in class.

Every other week I give students an in-class quiz during the second half of one of the lectures. These quizzes are short (three or four problems), but the problems require a little bit more thought than the previously described assignments. On a quiz after studying implicit differentiation, for example, students had the following problem:

> Find $\frac{d^2y}{dx^2}$ when x and y are related by the following equation:
> $$x^2 + xy + y^3 = 1.$$

This type of question requires a little bit more work and thought than the previously described types of assignments, and also helps me to realize which students have a good grasp of the material, and which students might need more help.

Alternating weeks with the quizzes, students have a longer written homework assignment due every other week. These assignments are always made available at least a week before being due because they contain reasonably difficult problems such as the following:

> Suppose that Alice and Bob both begin driving cars at 10:00pm Friday evening, and that Alice is initially 10 miles to the East of Bob. If Alice is driving North-East at a speed of $50\sqrt{2}$ miles per hour and Bob is driving North-West at a speed of $40\sqrt{2}$ miles per hour, how quickly is the distance between Alice and Bob increasing at 3:00am Saturday morning? (Hint: Since Alice is traveling North-East, her Eastward-speed and her Northward-speed are the same. Similarly, Bob's Westward-speed and his Northward-speed are the same.)

This type of problem requires several things from students: they first have to parse the question to determine what they're being asked to do, then interpret the set up of the problem mathematically, and in this particular problem students also have to realize their initial setup of the problem (as a quadrilateral with four sides changing at different speeds) is hard to work with, and then try to find another, simpler way to attack the problem.

Applications of the Theory. While I am a pure mathematician and believe that interesting problems are inherently worthy of study, I recognize that many students prefer to see the real-world applications of ideas we discuss in class. For this reason I try to highlight applications through examples in class, as well as in out-of-class projects and homework assignments. One example of an interesting, real-world application that I like to discuss in linear algebra is the PageRank algorithm which is used by Google to determine the "importance" of a web page, which in turn is used to determine the order of results in a Google search. After lecturing on eigenvectors and eigenvalues in class, I give students an out-of-class assignment where they must write some code in Sage to implement the PageRank algorithm and use it to determine the importance of fictional webpages in a graph that describes how these pages link to one another. This assignment gives me an excuse to discuss the Perron-Frobenius theorem in linear algebra as well as shows students a real-life example of an eigenvector/eigenvalue problem.