Teaching Statement
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Teaching mathematics effectively is a challenging, multifaceted endeavor that goes beyond simply relaying information in a lecture. The best teachers are able to not only engage their students while exposing them to difficult ideas, but also act as a mentor that helps students learn from mistakes, a coach that instills the confidence needed to succeed, and challenges students to hold themselves to a higher standard. I strive for this ideal by motivating each topic discussed in class with modern applications, giving students regular coursework with detailed feedback, and encouraging students to develop their own understanding.

Teaching philosophy and goals. My basic philosophy is that all students are capable of success and should be held to a high standard. However, I acknowledge that some students need extra encouragement and motivation; part of being good teacher means helping students feel that they can succeed and showing them why the material in class is useful, interesting, and worth taking the time to understand.

Besides teaching the material related to whatever course I am teaching, my main goals in teaching are to expose students to a larger mathematical world, encourage critical thinking, and help students become independent learners and problem solvers. There is a lot of interesting mathematics in the world that, unfortunately, most students will never see as part of their typical math courses. For this reason I make an effort to tell students about other mathematical ideas if there is a connection to what we discuss in class. For instance, when we discuss optimization in calculus, I tell the students about operations research and how it is used in many industrial settings; when we discuss inner products in linear algebra, I tell students that having an inner product gives us some basic geometric notions, and more advanced geometry uses inner products in a fundamental way.

While I have fun telling students about these additional mathematical topics, and I hope it catches the interest of at least a few students, I recognize that the vast majority of students will not be math majors, and even fewer (if any) will become professional mathematicians. In particular, I understand that the specifics of what we discuss in class will begin to fade from their memory once the semester ends. However, the general skills of how to think carefully and precisely, how to learn difficult and technical material, and the basic problem-solving strategies we discuss are more likely to remain with students and to be skills they can transfer to other areas.

How I teach. In order to motivate students and help them see the value in what we are discussing in class, I often present real-world applications of the material. For example, in calculus I mention applications to computer graphics and video games; in introductory probability courses I describe how Bayes' formula is used in machine learning; in linear algebra I discuss Google's PageRank algorithm when we learn about eigenvectors and eigenvalues; and in geometry I discuss how curvature provides the mathematical foundation of Einstein's theory of general relativity. Even though we may not have the time to go over each application in depth in class, the existence of these applications can help students to realize the utility in the ideas we are discussing and stay interested and motivated.

While motivation is important, the ultimate goal is to have students think deeply about the material, learn when certain theorems and ideas can be applied, and learn how to solve problems. I have found that this is best accomplished by incorporating active learning into my lectures, and also giving students regular coursework with detailed feedback on any errors so that students can learn from their mistakes.

After discussing the concepts underlying an idea in class and doing a few examples, I often have students work together in informal groups to solve problems and then volunteer to present their solutions to the class. While students may make mistakes in their presented solutions, their mistakes often highlight a common misunderstanding that I can discuss with the class as a whole. By gently reminding students that making mistakes is a natural part of the learning process, I find students quickly become comfortable volunteering to present their solutions, and seeing solutions with mistakes which we can correct on the spot becomes a valuable part of the class for many students.

The out-of-class assignments I give students contain a mix of conceptual problems and computational exercises, ranging from simple problems similar to in-class examples, to more challenging problems that force students to exercise their problem-solving skills. After the assignments have been graded I provide a key with detailed solutions and discuss any common mistakes during class.

I believe it is important to help students push themselves by giving at least a few challenging exercises with each assignment. While students are sometimes frustrated by difficult problems they can not immediately solve, I make myself available to students to help them and nudge them in the right direction when they get stuck. I hope that by pushing students to work on harder problems, they learn more and are better prepared not only for exams in my class, but are also more prepared for the more challenging classes they will take later. I regularly remind my students that it is okay to get stuck on a problem, and it is okay to make mistakes; the important thing is that they persevere and try to learn from the mistakes they do make.

Teaching diverse groups of students. Having taught a variety of courses ranging from low-level service courses and terminal courses to upper-level courses populated exclusively with math majors, at schools ranging from small liberal arts colleges to large research universities, I have had students from all walks of life in my classroom. Students come into the classroom with different expectations, experiences, and desired outcomes, and part of my job is to encourage and engage all of my students.

Part of the way I engage students and help them feel that they can be successful is by making an effort to ensure students feel comfortable talking to me in office hours and asking questions in class. By treating each student with respect, letting them know that I value them as individuals, and being cognizant of how my language (including my tone, body language, and facial expressions) may be interpreted, I try to present myself as friendly and non-threatening. My belief is that students will be more willing to discuss the material with me, any concerns they have, or what they believe is preventing them from performing their best in the class, if they recognize that I genuinely care about them and their success.

Continual improvement in teaching. While I have several years of experience teaching a variety of courses, I know that I have more to learn about teaching and can always seek to make incremental improvements in how I teach. To this end, I take feedback I receive from students and other faculty very seriously, adjusting my teaching accordingly. I have also been attending weekly pedagogy workshops at Bucknell, incorporating ideas presented at these weekly meetings, as well as ideas from faculty in other departments that arise in informal conversations. One example of something I have started doing after attending these workshops is to have anonymous, mid-semester evaluations. These give students an opportunity to tell me what they think of the course while there is still time to make adjustments that affect the students. In my calculus course this semester, for instance, many students commented that the active learning in class was one of the things they found the most beneficial, and as such I am making an effort to include more active learning exercises in the remainder of the semester.

Summary. In summary, I believe teaching is not simply about relaying facts, but also about exposing students to a larger mathematical world, inspiring and encouraging students to believe they are capable of success, and keeping students interested and motivated. This is not always an easy task, but one I pursue by using interesting examples, active learning in the classroom, challenging out-of-class exercises, and providing regular feedback. While no teacher is perfect, I am continually working towards becoming a better teacher with each passing year. The following pages provide an addendum which gives some specific examples of what I do in and out of the classroom, the types of exercises I assign, and how I work to support students who are struggling.

Addendum: Detailed Description of Teaching

In this addendum I provide some specific examples of how I support students while challenging them with difficult concepts, how I try to address common misconceptions, how I gradually push students from straight forward exercises to more involved problems, and how I try to interest students in mathematics as a whole.

Active learning. Students seem to learn best, and be much happier with the course, when they are actively engaged in the learning process. Instead of simply lecturing and have students take notes, I often ask the students for their opinions on what the next step of an example I am working on the board should be, and also have the students work on simple problems together during class. This is beneficial for me because it allows me to gauge in real-time how well the students are grasping new material, and it is also beneficial for the students because it makes the class more interesting and lively.

Technology. There are many technological tools available to help students visualize concepts from class, directly interact with data, and visualize mathematical objects. I use tools such as Wolfram Alpha, Mathematica, Sage, and Desmos in class when appropriate, and I tell the students about these tools so they can use them outside of class. (I am, however, very careful to make sure students understand that technology is a tool to help them learn the material, and not something that can do all of the work for them.) By taking advantage of technology when appropriate, we can spend more of class time working on developing students' conceptual understanding and problem-solving skills, and less time on rudimentary calculations.

Helping struggling students. While I believe that each student is ultimately in charge of deciding how successful they will be in course, I know some students will struggle for a variety of reasons. Part of my job is to help students get back on track when they struggle. Besides simply having regular office hours and trying to be reasonably quick to respond to e-mails, I make an effort to actively reach out to students that I believe are struggling. On a recent exam, for instance, I emailed each student that received a grade below a C to set up a one-on-one meeting in which we discussed how their exam score affects their final grade, and also discuss strategies for preventing low grades on future exams. For example, I ask the students how they are studying, both as a regular part of their daily schedule, and also how they changed their study practices when they knew an exam was coming up. I then make suggestions for how they could study more effectively, and also remind them that I am there to help them and they are always welcome and encouraged to discuss the course with me if any topic is giving them trouble.

My responsibilities as a teacher do not simply end when lecture ends. While I hope my students will be independent and willing to work hard to understand the material discussed in class and complete the assigned exercises, I realize that sometimes students need additional help. For this reason, I make a point to be patient and understanding with students, addressing any concerns they have about the material in the course or stumbling blocks they have encountered while working on an assignment. I do my best to provide students with alternative explanations of topics or counterexamples to any erroneous logic coming from a student's misconception.

In probability, for example, I once had a student state that they did not understand why the law of total probability was necessary for calculating the probability of pulling a certain colored marble from one of two urns with different numbers of marbles: the student did not understand why this was different from thinking of having all of the marbles together in one urn. To show the difference between the two types of problems, I told the student to imagine a situation where one urn contained one black marble and a second urn contained ninety-nine white marbles. If you randomly select an urn, then randomly select a marble from that urn, the probability you select a black marble is $1/2$ since the color of marble is completely determined by the urn you select. If you had one giant urn with one black marble and ninety-nine white marbles, however, the probability of selecting a black marble is $1/100$.

Along these same lines, I often show students not only the correct way to solve a problem in class, but also a common incorrect way. After "solving" the problem with both methods and seeing that we obtain different answers, we have a class discussion about the distinction between the two methods: why does one method work and not the other? I believe that seeing these kinds of incorrect examples and the ensuing discussion helps students to become more focused in their reasoning, showing them that their naive intuition can lead them astray if they're not careful.

In courses I have taught multiple times, I provide students with a typed copy of my lecture notes that contain not only the examples from class, but also the detailed derivations that are often too time-consuming for lecture, and several practice problems with detailed solutions. These notes allow students to pay more active attention in lecture since they do not need to worry as much about writing down every detail. While preparing notes like this can be laborious, the students often say in end-of-semester evaluations that these notes are one aspect of my classes they find the most helpful. Each time I re-teach the class I update these notes correcting any previously unnoticed typos, adding more practice problems, and offering further explanations to any topics students had trouble with the last time I taught the class.

Addressing Misconceptions. Many students in calculus struggle with the idea of a limit, and in particular in understanding the difference between taking the limit of a function as x approaches a and evaluating the function at $x = a$. When I anticipate that students will have these types of common misconceptions, I first address them in class through examples, and then give the students homework assignments where they have to face these misconceptions head-on. The following is an example of a homework problem I gave in a first-semester calculus course:

> Construct a pair of functions $f(x)$ and $g(x)$ such that $f(x)$ and $g(x)$ are defined everywhere, and the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist for all a, but
> $$\lim_{x\to 3} f(x)g(x) \neq f(3)g(3).$$

This problem essentially asks students to come up with a function which is discontinuous at $x = 3$, but many students come to office hours initially believing it is impossible to construct a pair of functions as described in the problem. After talking students through the problem, they eventually have an "Ah-ha!" moment when the relationship between a limit and evaluating a function at a point starts to make more sense, and they appreciate why continuous functions are desirable.

Gradual Challenge. There are a variety of reasons why a given student may be averse to challenging themselves, but learning any new or difficult concept inherently requires challenge. In order to help my students build confidence and be more open to challenge, I assign regular exercises that gradually push students a little bit more with each assignment.

Every other week I give students an in-class quiz during the second half of one of the lectures. These quizzes are short (three or four problems), but the problems require a little bit more thought than the previously described assignments.

Alternating weeks with the quizzes, students have a longer written homework assignment due every other week. These assignments are always made available at least a week before being due because they contain reasonably difficult problems such as the following:

> Suppose that Alice and Bob both begin driving cars at 10:00pm Friday evening, and that Alice is initially 10 miles to the East of Bob. If Alice is driving North-East at a speed of $50\sqrt{2}$ miles per hour and Bob is driving North-West at a speed of $40\sqrt{2}$ miles per hour, how quickly is the distance between Alice and Bob increasing at 3:00am Saturday morning? (Hint: Since Alice is traveling North-East, her Eastward-speed and her Northward-speed are the same. Similarly, Bob's Westward-speed and his Northward-speed are the same.)

This type of problem requires several things from students: they first have to parse the question to determine what they're being asked to do, then interpret the setup of the problem mathematically. In this particular problem, students also have to realize their initial setup of the problem (as a quadrilateral with four sides changing at different speeds) is hard to work with, and then try to find another, simpler way to attack the problem.

A Larger Mathematical World. When appropriate, I like to show my students the material we are learning in class is part of a larger mathematical story, in the hopes of piquing their interest in mathematics as a whole. In numerical analysis, for example, after the students implement Newton's method in Matlab, I discuss a surprising connection to complex dynamics and fractals. When restricted to real numbers, Newton's method can fail to converge for simple polynomials. If we use complex numbers, however, the fundamental theorem of algebra guarantees the existence of roots. By interpreting the (x, y)-coordinates of each pixel on the screen as the real and imaginary parts of a complex number, we can generate an image by coloring the pixel according to where it converges after iterating Newton's method several times. This is an example of a fun application that surprises students and which I hope inspires some of the students to go online after class, visit Wikipedia, and learn more about a new area of mathematics on their own.